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Effect of fiber layout and stacking sequence on free vibration response of the symmetric laminated composite plate.

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Abstract - The objective of this work is to examine the effect of fiber layout and stacking sequence on the dynamic response of a carbon fiber reinforced composite plate. Finite element (FE) software: Abaqus was deployed for the numerical investigation and the findings were compared with the Ritz method. A rectangular plate (450 $\times$ 300 mm^2) having a thickness of 0.762 mm was selected for this study. The analysis expresses that the fiber layout and stacking sequence significantly influence the vibration frequencies and corresponding mode shapes.

Keywords—Natural Frequencies, Mode shapes, Carbon fiber reinforced composite plate, Fiber layout, Stacking sequence.

I. INTRODUCTION

It is commonly seen that long fibers exhibit more stiffness and strength compared to the same material in the bulk form from which they are made. The main reason for this is less number of defects in the fibers than in bulk materials as fibers have near crystal sized diameter [1]. Almost all the structural components and machine elements are exposed to cyclic loading in their service life. Therefore, effect of vibration is very prominent whether it is small in amplitude or large on the performance. It is thus necessary to have prior knowledge of the response of the structural/ machine elements when subjected to dynamic loading.

Generally, CFRP plates may have various orientations of fibers and stacking sequences of laminas. The change in the layout of fibers and stacking sequence in a CFRP plate with fixed boundaries give rise to change in stiffness of CFRP plate. This affects the vibration characteristics of the plate. Many theories and methods to perform vibration analysis of composite plates are available in literature, [2-3] and many authors have used these theories in their work [4-8]. Hearmon [10] proposed the use of beam characteristics shape corresponding to a beam fixed on each end for a plate with fixed edges. Thai and Kim [11] deployed the refined plate theory to execute a free vibration analysis of orthotropic plates. Bhatt and Dwivedi [12] used shell elements in ANSYS to perform vibrational analysis on symmetric cross-ply laminated plate. Jungian et. al. [13] reported that raft frames made from composite material resulted in more passenger comfort in comparison to steel frames because of enhanced damping. Lee et. al. [14] reported that the vibration response of unidirectional composite plate depends on fiber layout.

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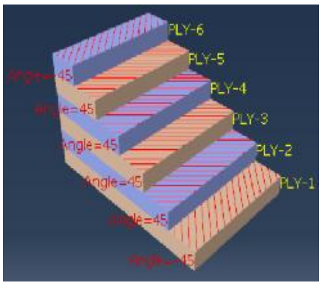
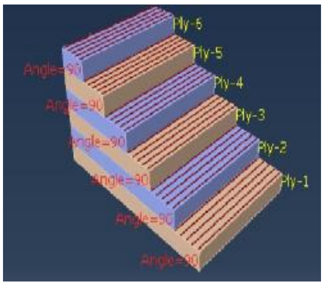
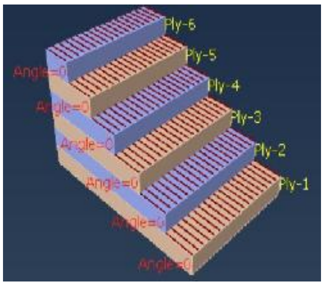
In this article, natural frequencies and mode shapes of AS4/3501-6 carbon epoxy composite plate computed through FE software: Abaqus is compared with Ritz method. Fig 1 shows different fiber layouts and stackingsequences used in this study. The results show that for the structural design of fiber reinforced laminated plates the fiber angle and stacking sequence play an important

(b)





role in



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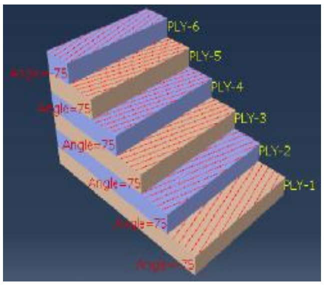
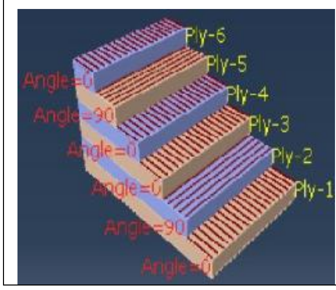


characteristics(a)

(c)

(d)

(e)



(f)

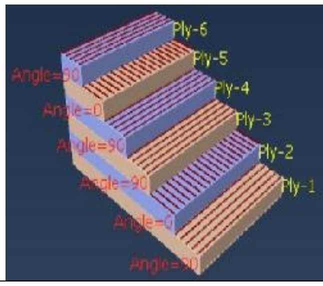


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Fig 1. Sets of CFRP plates: (a) $\left[0^\circ \quad 0^\circ \quad 0^\circ\right]_S$, (b) $\left[-45^\circ \quad 45 \quad 45\right]_S$, (c) $\left[-75^\circ \quad 75 \quad 75\right]_S$, (d) $\left[90^\circ \quad 90^\circ \quad 90^\circ\right]_S$, (e) $\left[0^\circ \quad 90^\circ \quad 0^\circ\right]_S$ (f) $\left[90^\circ \quad 0^\circ \quad 90^\circ\right]_S$

II. MATHEMATICAL FORMULATION:

Consider a thin symmetric fiber reinforced composite plate with dimensions a, b, and H in the x, y, and z directions. The plate is made of k number of layers of fiber reinforced material called laminas, Fig 2 shows the geometry of plate. Plate is fixed at all four edges i.e.; fixed boundary conditions are used. As in the case of free vibration, there is no external transverse distributed load applied to the plate.

Fig. 2. Geometry of a laminated plate.

Assumptions of the classical laminate theory which is the most common plate theory is used in the analysis of laminar composites.

The appropriate energy criterion governing free vibration response of laminated plates with no lateral and inplane loads $\left(q(x,y)=N_x=N_y=N_{xy}=0\right)$ is [15]:

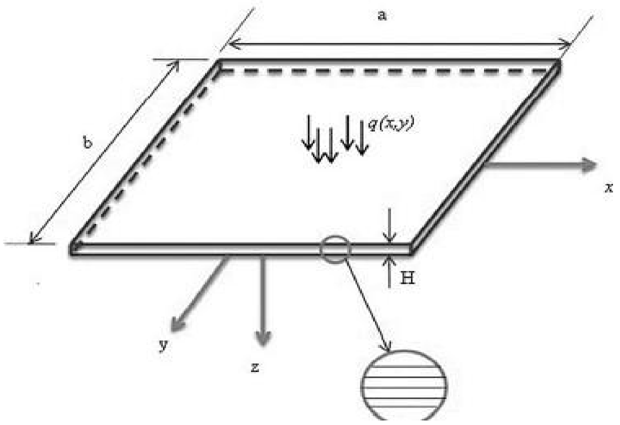
$$U-T=\frac{1}{2}\int_0^b\int_0^a [D_{11}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+2D_{12}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+D_{22}\left(\frac{\partial^2 w}{\partial y^2}\right)^2+$$

$$4D_{66}\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2-\circ\mu^2 w]dxdy \tag{1}$$

Where, $\circ = \int_{-H/2}^{H/2} \circ_0^{(k)} dz$ (2)

And $D_{ij}=\int_{-H/2}^{H/2} \overline{Q}_{ij}^{(k)} z^2 dz$ (3)

According to the Ritz method, we assume the solution [15]



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$$w(x,y)=\sum_{m=1}^{M}\sum_{n=1}^{N}A_{mn}X_m(x)Y_n(y) \qquad (4)$$

Using this solution in the derivative of equation (1), we obtain the homogeneous algebraic equations:

$$\sum_{I=1}^{M}\sum_{j=1}^{N}\{D_{11}\int_0^a \frac{\partial^2 X_i}{\partial x^2}\frac{\partial^2 X_m}{\partial x^2}dx\int_0^b Y_jY_n dy\ +$$
$$D_{12}\left[\int_0^a X_m\frac{\partial^2 X_i}{\partial x^2}dx\int_0^b Y_j\frac{\partial^2 Y_n}{\partial y^2}dy+\int_0^a X_i\frac{\partial^2 X_m}{\partial x^2}dx\int_0^b Y_n\frac{\partial^2 Y_j}{\partial y^2}dy\right]$$
$$+D_{22}\int_0^a X_iX_m dx\int_0^b\frac{\partial^2 Y_j}{\partial y^2}\frac{\partial^2 Y_n}{\partial y^2}dy+4D_{66}\int_0^a\frac{dX_i}{dx}\frac{\partial X_m}{\partial x}dx\int_0^b\frac{dY_j}{dx}\frac{\partial Y_n}{\partial y}dy-$$
$$°\mu_{mn}^2\int_0^a X_iX_m dx\int_0^b Y_jY_n dy\}A_{ij}\ =\ 0$$

(5)

Since the system of Equations in equation (5) is homogeneous, a nontrivial solution which is natural frequency μ_{mn} is determined only if determinant of the coefficient matrix is zero.

Hearmon [10] has suggested, $X_m(x)$ and $Y_n(y)$ for each mode, for plates with fixed edges:

For edges x=0, a

$$X_m(x)=¥_m\cos\frac{a_m x}{a}-¥_m\cosh\frac{a_m x}{a}+\sin\frac{a_m x}{a}-\sinh\frac{a_m x}{a} \qquad (6)$$

For edges y=0, b

$$Y_n(y)=¥_n\cos\frac{a_n y}{b}-¥_n\cosh\frac{a_n y}{b}+\sin\frac{a_n y}{b}-\sinh\frac{a_n y}{b} \qquad (7)$$

Where a_m, a_n are the roots of the equation (8) :

$$\cos a_i \cosh a_i=1 \qquad (8)$$

And

$$¥_i=\frac{\cos a_i-\cosh a_i}{\sin a_i+\sinh a_i} \qquad (9)$$

Using equation (6) And (7), the equation (5) can be simplified as:

$$\mu_{mn}=\frac{1}{a^2\sqrt{°}}\sqrt{D_{11}\sim_1^4+2(D_{11}+2D_{66})R^2\sim_2+D_{22}R^4\sim_3^4}$$

(10)

Where, $\sim_1,\sim_2$ and $\sim_3$ are tabulated in the Table (1) and R being the aspect ratio (a/b).

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Table 1 [15]: Values of $\sim_1$,$\sim_2$ and $\sim_3$ for all sides fixed boundary conditions.

Boundary Conditions	$\sim_1$	$\sim_2$	$\sim_3$	m	n
All sides fixed	4.730	4.730	151.3	1	1
	4.730	$(n+0.5)$®	$12.30 \sim_3 (\sim_3 - 2)$	1	2, 3, 4...
	$(m+0.5)$®	4.730	$12.30 \sim_1 (\sim_1 - 2)$	2, 3, 4...	1
	$(m+0.5)$®	$(n+0.5)$®	$(\sim_1 \sim_3 (\sim_1 - 2)(\sim_3 - 2))$	2, 3, 4...	2, 3, 4...

III. Numerical Simulation

In the Abaqus software using the Lanczos eigenvalue extraction method the numerical model analysis of rectangular CFRP plates made of 6 laminas, each having an average thickness h=0.127 mm and mass density $°_o$=1600 kg/m^3 with different sets of fiber orientation as shown in Figure 1 was performed. The encastre ($U_1 = U_2 = U_3 = UR_1 = UR_2 = UR_3 = 0$) boundary

condition at the edges of plates was considered. Shell elements (S4R) with four nodes are used to mesh the plates. The area of the rectangular composite plate is 450 × 300 mm^2. The material properties of lamina used for simulation are as follows [16]:

E_1 = 147.00 GPa, E_2 = 10.30 GPa,

G_{12}=7.00 Gpa, G_{13}=7.00 GPa, G_{23}=3.70 GPa,

$¬_{12}$ = 0.27, $¬_{23}$ = 0.57, $¬_{13}$ = 0.27

IV. RESULTS AND DISCUSSIONS:

In this section, a discussion on various results obtained from the analysis is presented.

A. *Natural frequencies of rectangular plates:*

First modal analysis was executed with finite element software: Abaqus as discussed in section (3). An analytical analysis using equation (10) was also performed in order to authenticate results obtained from Abaqus. The modal indices (m, n) obtained from the numerical modal analysis are further used in analytical observation to find the natural frequencies for respective mode shapes.

Comparison between numerical results and analytical results for the different sets of fiber orientations is presented in Table 2 and Table 3. The error, which is calculated using $(|f_A - f_N| / f_A) \times 100$,where f_A is natural frequency obtained by analytical solution and f_N is natural frequency obtained by numerical solution, does not exceed 5%. This shows that Abaqus can yield convergent and precise outcome.

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Mode	Solution Type	Natural frequency (Hz) and modal indices (m, n)			
		$\left[0^\circ \quad 0^\circ \quad 0^\circ\right]_S$	$\left[-45^\circ \quad 45 \quad 45\right]_S$	$\left[-75^\circ \quad 75^\circ \quad 75^\circ\right]_S$	$\left[90^\circ \quad 90^\circ \quad 90^\circ\right]_S$
First	Analytical	45.577 (1, 1)	63.962 (1, 1)	82.433 (1, 1)	85.370 (1, 1)
	Numerical	45.637 (1, 1)	62.619 (1, 1)	82.133 (1, 1)	85.444 (1, 1)
	Error	0.131%	2.099%	0.363%	0.086%
Second	Analytical	76.669 (1, 2)	104.180 (2, 1)	94.367 (2, 1)	92.184 (2, 1)
	Numerical	76.835 (1, 2)	100.270 (2,1)	93.451 (2, 1)	92.290 (2, 1)
	Error	0.215%	3.753%	0.971%	0.114%
Third	Analytical	108.423 (2, 1)	150.868 (1, 2)	115.976 (3, 1)	107.157 (3, 1)
	Numerical	108.490 (2, 1)	148.810 (1, 2)	114.520 (3, 1)	107.370 (3, 1)
	Error	0.061%	1.364%	1.255%	0.199%
Fourth	Analytical	130.834 (2, 2)	162.693 (3, 1)	148.268 (4, 1)	132.682 (4, 1)
	Numerical	131.060 (2, 2)	155.250 (3, 1)	146.290 (4, 1)	133.030 (4, 1)
	Error	0.173%	4.574	1.334%	0.262%
Fifth	Analytical	131.703 (1, 3)	200.052 (2, 2)	191.179 (5, 1)	169.321 (5, 1)
	Numerical	132.140 (1, 3)	194.490 (2, 2)	188.750 (5, 1)	169.890 (5, 1)
	Error	0.331%	2.780%	1.271%	0.335%
Sixth	Analytical	176.178 (2, 3)	238.676 (4, 1)	221.301 (1, 2)	216.647 (6, 1)
	Numerical	176.760 (2, 3)	226.470 (4, 1)	220.940 (1, 2)	217.610 (6, 1)
	Error	0.330%	5.114%	0.163%	0.444%

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From Table 2 it is observed that the frequency of the mode (1, 1) goes up when the fibre angle increases whereas the subsequent frequency of the mode (2, 1) goes down with an increase in fiber angle. $\left[90^\circ\ \ 90^\circ\ \ 90^\circ\right]_S$ Plate has maximum first natural frequency and $\left[0^\circ\ \ 0^\circ\ \ 0^\circ\right]_S$ has minimum. The natural frequencies of the $\left[90^\circ\ \ 0^\circ\ \ 90^\circ\right]_S$ plate show a significant difference from the natural frequencies of $\left[0^\circ\ \ 90^\circ\ \ 0^\circ\right]_S$ plate shown in table 3.

Table 2: Natural Frequencies of CFRP Rectangular plate obtained from analytical and numerical solutions.

Table 3: Natural Frequencies of CFRP Rectangular plate obtained from analytical and numerical solutions.

Mode	Solution Type	Natural frequency (Hz) and modal indices (m, n)	
		$\left[0^\circ\ \ 90^\circ\ \ 0^\circ\right]_S$	$\left[90^\circ\ \ 0^\circ\ \ 90^\circ\right]_S$
First	Analytical	58.553 (1, 1)	77.059 (1, 1)
	Numerical	58.640 (1, 1)	77.151 (1, 1)
	Error	0.148%	0.119%
Second	Analytical	104.455 (2, 1)	96.655 (2, 1)
	Numerical	104.600 (2, 1)	96.811 (2, 1)
	Error	0.138%	0.161%
Third	Analytical	135.446(1, 2)	139.683 (3, 1)
	Numerical	135.700(1, 2)	139.970 (3, 1)
	Error	0.187%	0.205%
Fourth	Analytical	165.496 (2, 2)	203.719 (1, 2)
	Numerical	165.850 (2, 2)	204.020 (1, 2)
	Error	0.213%	0.147%
Fifth	Analytical	185.537 (3, 1)	206.063 (4, 1)
	Numerical	185.850 (3, 1)	206.590 (4, 1)
	Error	0.168%	0.256%
Sixth	Analytical	230.989 (3, 2)	215.562 (2, 2)
	Numerical	231.520 (3, 2)	215.930 (2, 2)
	Error	0.229%	0.171%

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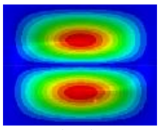
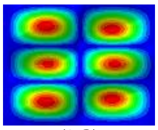
Lay out →	$\begin{bmatrix} 0^\circ & 0^\circ & 0^\circ \end{bmatrix}_S$	$\begin{bmatrix} -45^\circ & 45 & 45 \end{bmatrix}_S$	$\begin{bmatrix} -75^\circ & 75^\circ & 75^\circ \end{bmatrix}_S$	$\begin{bmatrix} 90^\circ & 90^\circ & 90^\circ \end{bmatrix}_S$
Mode 1	(1,1)	(1,1)	(1,1)	(1,1)
Mode 2	 (1,2)	(2,1)	(2,1)	(2,1)
Mode 3	(2,1)	(1,2)	(3,1)	(3,1)
Mode 4	(2,2)	(3,1)	(4,1)	(4,1)
Mode 5	(1,3)	(2,2)	(5,1)	(5,1)
Mode 6	 (2,3)	(4,1)	(1,2)	(6,1)

Fig (3) shows the changes that arise in mode shapes due to changes in fiber layout. It is seen that for all the fiber orientations the mode (1, 1) comes out as the first mode of vibration. Whereas from the second mode variation appears, for $\begin{bmatrix} 0^\circ & 0^\circ & 0^\circ \end{bmatrix}_S$ plate mode (1, 2) emerges as second mode, for $\begin{bmatrix} -45^\circ & 45 & 45 \end{bmatrix}_S$ plate mode (2, 1) emerges as second mode and mode (1, 2) emerges as third mode. For $\begin{bmatrix} -75^\circ & 75^\circ & 75^\circ \end{bmatrix}_S$ plate sixth mode is mode (1, 2) whereas for $\begin{bmatrix} 90^\circ & 90^\circ & 90^\circ \end{bmatrix}_S$ plate mode (6, 1) emerges as sixth mode.

Fig (3): Comparison Mode shapes of rectangular (aspect ratio = 1.5) fiber reinforced laminar plates having different fiber layout.

V. Conclusion:

In this work, a theoretical and numerical investigation is done to find the impact of the fiber layout and stacking sequence on natural frequencies and mode shapes of the rectangular carbon-epoxy laminated plate. Relative error between theoretical solution obtained by the Ritz method using assumptions of classical laminate theory and the numerical solution obtained by Finite element software does not exceed 5 %. The natural frequencies changes when there is a change in the length of fibers in the composite plate due to change in fiber orientation angle or due to change in stacking sequence because this alters the stiffness of composite plate. It was seen that the natural frequency of mode (1, 1) which emerges as first mode of vibration in case of all sets of plates increases with the increase in fiber orientation angle whereas for other modes it increases or decreases. The present paper concludes that we can vary natural frequencies and corresponding mode shapes according to the need of application by changing the fiber layout and stacking sequences of laminas.

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