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Efficient Image Segmentation Methods Based On Otsu And Kapur For Natural Images Applications

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***Abstract*—Image segmentation, the process of dividing an image into meaningful regions, underpins numerous applications in computer vision. This review delves into the diverse approaches adopted for image segmentation, analyzing their strengths and limitations. Image segmentation is the primary step for several computer vision problems, which is the process of partitioning an image into constituent parts or regions in such a way that all the regions are homogeneous. Ideally, the goal of segmentation should be to produce regions that correspond to distinct objects in the image. The segmentation techniques are broadly categorized into thresholding, region-based, edge-based, and deep learning methods. Out of various methods, thresholding is effective in finding segmentation. Thresholding methods, distinguished by their simplicity and computational efficiency, are susceptible to noise and uneven illumination. In this scheme, multiple thresholds are computed on the histogram and categorize the pixels from an image into dissimilar classes or regions based on threshold levels; in the bi-level thresholding method, one threshold is selected to segment the image into two regions only. By maximizing the inter-class variance, optimal threshold levels can be computed with various optimized techniques. To Compute the optimized threshold values, Otsu's variance and Kapur's entropy are deployed as fitness functions, both the values should be maximized to locate optimal threshold values. In both Kapur's and Otsu's methods, the pixels of an image are classified into different classes based on the threshold level selected on the histogram. Our analysis emphasizes the importance of tailoring the segmentation technique to the specific application and image characteristics by comparing Otsu's method and Kaur's method for segmentation.**

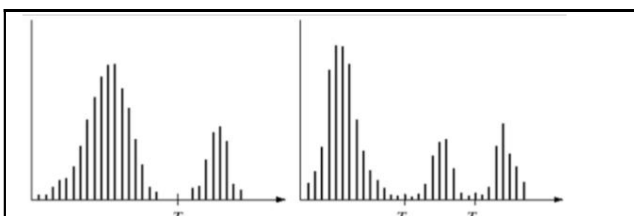
Keywords—Image segmentation, object detection, computer vision, multilevel threshold, Otsu and Kapur Methods.

I. INTRODUCTION

Image segmentation, the process of partitioning an image into meaningful regions, is a fundamental task in computer vision with applications in diverse fields like medical imaging, robotics, self-driving cars, and content-based image retrieval[2]. Over the years, numerous segmentation methods have emerged, each with its own strengths, limitations, and applicability. This review delves into various segmentation techniques, covering thresholding, edge-based, region-based, energy-based, and active contour models, exploring their theoretical foundations, application areas, potential drawbacks, and recent advancements. In various sectors and real-world applications such as military, medical, astronomy, etc. pattern recognition, image analysis, and image disciplines are the most important subjects in computer science and computer engineering [1,2]. Image segmentation is an important and difficult process [3]. From a multi-media perspective, image segmentation can be applied to a single image or a series of images that form a video [1,4,5]. Although there exists a substantial body of literature on image segmentation and a plethora of methods have been employed, segmentation can broadly be achieved through various approaches, including thresholding [6-7], edge-based segmentation [8,9], region-based segmentation [10-13], and energy-based segmentation [14-16]. As such, the subsequent subsections will commence with an examination of thresholding-based segmentation, followed by an exploration of the remaining three categories.

II. MULTILEVEL THRESHOLDING

Predefined threshold. Its efficiency makes it appealing for real-time tasks. Global thresholding uses a single threshold for the entire image, suitable for uniform scenes with high-contrast objects. Adaptive thresholding employs dynamic thresholds based on local image statistics, enabling adaptation to varying intensities. Otsu's method, a variant, automatically computes the optimal threshold. Advantages include computational speed, ease of implementation, and suitability for grayscale images with clear contrast. However, limitations like difficulty handling noise, complex objects, and low-contrast scenarios restrict its generalizability.



Single Threshold	Multiple Threshold

Fig. 1. Histogram for multilevel thresholding to segmentation of the image

The most used strategy for segmentation is the thresholding technique. In the thresholding process, image pixels were partitioned with the help of intensity level or the force of an image. This strategy is generally utilized to perceive forefront images from a background image. Even though it is a simple one and additionally fast, it doesn't think about the spatial normal for an image. Apart from these advantages, this thresholding strategy is extremely delicate towards noise and intensity in-homogeneity. In bi-level thresholding, there was only one threshold T, if the spatial point of an f(x, y) is larger than the threshold value T, i.e., f(x, y) > T is treated as an object point (pixel); otherwise, a pixel point (x, y) belongs to the background of an image if f(x, y) < T. Based upon the discussion previously mentioned, thresholding is the process perceived as an operation that endeavors to gain threshold T by use of the below equation.

$$=(,...) \quad (1)$$

A. Otsu Method

This technique[2-4] is used for Multi-Level thresholding (MT), in which gray levels will be partitioned into different regions or classes, in this process thresholding (th) levels to be selected, the set of rules to be followed for bi-level thresholding are

$$1 \leftarrow 0 \leq < \hbar, \qquad 2 \leftarrow \quad \hbar \leq < -1 \quad (2)$$

where C1 and C2 are two classes, p indicates the pixel value for the gray levels 1, 2, 3, ……, L − 1 in an image, and L − 1 has indicated the maximum gray level. If the gray-level below threshold th then that pixel is grouped into class C1, else grouped to class C2. The set of rules for in Multi-Level thresholding (MT) are

$$1 \leftarrow \quad 0 \leq < \hbar 1 \quad (3)$$
$$2 \leftarrow \quad \hbar 1 \leq < \hbar 2$$
$$\leftarrow \quad \hbar \leq < \hbar + 1$$
$$\leftarrow \quad \hbar \leq < \hbar + 1$$

from Equation.3, 1, 2, …… indicates different classes, and threshold levels to find objects represented by $\hbar 1, \hbar 2, \ldots, \hbar, \hbar$ + 1, $\hbar$, these thresholds can be computed based on a histogram. By use of these threshold levels, the entire pixels will be classified into different classes or exclusive regions. The significant methods of segmentation of images based on threshold levels are Otsu's and Kapur's methods, and in both cases, threshold levels can be computed by maximizing the cost function (inter-class variance). In this chapter, optimized threshold levels $\hbar$ for computed by Otsu's method $\hbar$values [16]. In this method, inter-class variance is considered as the objective function, also called a cost function. For experimentation, grayscale images are considered. The below expression gives the probability distribution for each gray-level

From Equation (3), pixel value denoted by , range of grayscale is(0 ≤ ≤ − 1), where = 1, 2, 3 for RGB and = 1 for grayscale image and total image pixels represented

$$\hbar = \frac{\hbar}{}, \quad \hbar = 1 \quad (4)$$

by NP, the histogram of considered images represented by $\hbar$. In bi-level thresholding, the total pixels in the image are grouped into two classes.

$$1 = \frac{\hbar}{(\hbar)}, \ldots, \frac{\hbar}{(\hbar)}, \qquad 2 = \frac{\hbar_{\hbar+1}}{(\hbar)}, \ldots, \frac{\hbar}{(\hbar)} \quad (4.1)$$

Whereas $_0 \hbar$ and $_1 \hbar$ are the probabilities distributions for 1 and 2, as it is shown below as

$$\hbar_0 = \sum_{=1}^{\hbar} \hbar, \qquad \hbar_1 = \sum_{=\hbar+1} \hbar \quad (5)$$

the mean of two classes 0 and 1 to be computed, the variance between classes ^{2} given by Equations (4) and Equation (5)

Notice that for both Equations.6 and 7, determined by the type of image, where and In Equation.7 the

$$\hbar \; \frac{\hbar}{} \ldots \frac{\hbar}{} \quad (6)$$

variances of classes 1 and 2 are given in Equation .8

$$^2 \qquad ^2 \quad (7)$$

Where the and = 1. Based on the values and , Equation(8) presents the objective function:

$$\hbar = \max {}^2 \; \hbar, \quad 0 \leq \hbar \leq -1 \quad (8)$$

From Equation.8 $^2 \hbar$ is the total variance between two various regions after segmentation by Otsu's scheme for given $\hbar$, optimization techniques are required to find the threshold level (th) by maximizing the fitness function as shown in Equation.8. Similarly for multi-level thresholding (MT), the objective (or fitness) function (th) shown in Equation.10 to segment an image into classes, requires variances.

= max th ,0≤th≤ −1, (9)

where = 1,2 ...

Where TH is a vector,= th_1, th_2, th_3 ... ... th −1 for multi-level thresholding, the variances between

(10)

classes can be computed from Equation.11

where th represents class, indicates probability of th classes and is the mean of ith class. For MT segmentation, these parameters are anticipated as below.

th_1

th = th

= th ... th (11)

= th

And, the averages of each class can be computed as

(12)

B. Multilevel Thresholding with Kapur Method

One more important nonparametric technique that is used to compute the optimal threshold values is Kapur's method[2-7], entropy as an objective function. This method focuses on finding the optimal thresholds by maximizing the overall entropy. The entropy measures the compactness and separability between classes. For the multilevel, the objective function of Kapur's method is defined as

= max $\sum_{i=1}^{k} H_i^C$, 0 ≤ ≤ − 1, where (13) = 1,2...

Where TH is a vector, TH = th_1, th_2, th_3 ... ... th_{k-1} . Each entropy is calculated separately with its th value, given for k entropies.

(14)

Ph^c_i is the probability distribution of the particular intensity levels which is obtained using (4). The values of the probability occurrence (w_0^c , w_1^c , w_2^c ...... w_{k-1}^c) of the classes are obtained using (14). In the end, by using Equation 3. classify the pixels into various classless.

III. OPTIMIZATION TECHNIQUES

In the course of computing the optimized threshold level for the segmentation of a image optimization techniques can be used to maximize the inter-class variance. A variety of evolutionary and bio-inspired techniques are developed, like Differential Evolution (DE) [17-18], Simulated Annealing (SA) [19], Tabu Search (TS) [20], PSO [19-20], Artificial Bee Colony (ABC) [21], HAS [21] and so on, in this thesis Electromagnetism-like Algorithm [23] is also used (EMO used on Energy Curve in place of the histogram). A wide variety of evolutionary algorithms such as Differential Evolution, Simulated Annealing, Tabu Search, etc., for Otsu's method to compute threshold levels [23-25], and Genetic Algorithms described to segment multi-classes methods [26-27]. This EMO method mimics the electromagnetism law of physics, and it can be used efficiently to solve global optimization problems [26]. In this paper HAS usd to find optimized threshold levels with Otsu's and Kapur's methods.

IV. COMPARATIVE MEASURES

A. Dunn's indices (DI)

Dunn's Index discloses two characteristics of clustering methods which are (i) low intra-cluster distances (ii) high inter-cluster distances. The DI is a ratio of the minimal inter-cluster distance $\min_{x \in C_i,\, y \in C_j} d(x, y)$ and maximal intra-cluster distance $\max_k \max_{x,y \in C_k} d(x, y)$

= (15)

The higher value of DI indicates effective clustering. From Equation (15) of DI C_i is i_{th} cluster in the image, x & is pixel gray levels, the distance between pixels x and y is d(x, y).

B. *Peak-to-Signal Noise Ratio (PSNR)*

PSNR gives the deviation between the segmented image and the original. For better segmentation, its value should be high. The PSNR can be computed from root-mean-square error (RMSE) as given in Equation 16.

$$=20 \ 10 \quad \frac{255}{} \quad \frac{}{0 \times 0} \qquad (16)$$

From the above equation, I(i, j) is the input image, IS i, j is the final segmented image. The C0 denoted the number of columns and R_0 indicates the number of rows of an image considered for segmentation.

V. RESULTS ANALYSIS

The results of comparing Otsu's method and Kapur's method for image segmentation across different threshold levels and images are shown in terms of PSNR (Peak Signal-to-Noise Ratio) and DB Index (Davies–Bouldin Index). The performance of both methods is analyzed for four different images: Bridge, Cameraman, Lena, and Butterfly, at threshold levels (N) of 2, 3, 4, and 5.

For the Bridge image, as the threshold level increases from 2 to 5, both methods show a steady increase in PSNR. At N = 2, Otsu's method achieves a PSNR of 10.5823, while Kapur's method yields 9.6129. This difference persists, and by N = 5, Otsu's method reaches a PSNR of 18.4329, whereas Kapur's method attains 18.4931. In terms of the DB Index, Otsu's method performs better, with values starting at 0.2259 at N = 2 and decreasing to 0.209 at N = 5. In contrast, Kapur's method has a higher DB Index, ranging from 0.2569 at N = 2 to 0.201 at N = 5.

For the Cameraman image, Otsu's method consistently outperforms Kapur's method in PSNR. At N = 2, Otsu's PSNR is 12.351, while Kapur's method only achieves 6.1718. This significant difference continues at N = 5, with Otsu's method reaching a PSNR of 21.533, compared to 20.1531 for Kapur's method. Regarding the DB Index, Otsu's method also shows superior performance, with values ranging from 0.5524 at N = 2 to 0.2445 at N = 5, while Kapur's method starts with a much lower DB Index at 0.0768 at N = 2 and ends with 0.2354 at N = 5.

In the case of the Lena image, the two methods perform similarly in terms of PSNR. For instance, at N = 2, Otsu's method has a PSNR of 12.0374, while Kapur's method scores 11.8660. At N = 5, both methods produce almost equal PSNR values, with Otsu's method at 18.6581 and Kapur's method at 18.1066. The DB Index for both methods is comparable, with Otsu's method starting at 0.2189 at N = 2 and reducing to 0.167 at N = 5, whereas Kapur's method starts at 0.2196 and ends at 0.1559, showing a slightly better performance at the higher threshold.

For the Butterfly image, Otsu's method again achieves higher PSNR values at all threshold levels. At N = 2, Otsu's method has a PSNR of 10.9124, compared to 6.2464 for Kapur's method. By N = 5, Otsu's PSNR increases to 17.4215, while Kapur's method reaches 16.7266. The DB Index shows Otsu's method performing better with values ranging from 0.2954 at N = 2 to 0.2986 at N = 5. Kapur's method, meanwhile, has DB Index values of 0.2871 at N = 2, increasing slightly to 0.2681 at N = 5.

Overall, Otsu's method demonstrates better performance in terms of PSNR and DB Index across the images. The average PSNR for Otsu's method is 15.51, compared to Kapur's average of 14.39. Similarly, the average DB Index for Otsu's method is lower at 0.2396, while Kapur's method has an average DB Index of 0.2055, suggesting better segmentation with Otsu's method.

In terms of threshold levels (Table II), for the Bridge image at N = 2, Kapur's method selects a threshold of 116, whereas Otsu's method selects 127. As the number of threshold levels increases, Kapur's method tends to select slightly lower thresholds overall, with values of 53, 98, 147, and 199 at N = 5, compared to Otsu's selections of 65, 105, 144, and 192.

For the Cameraman image, Kapur's method selects significantly higher thresholds than Otsu's method, especially at N = 2, where Kapur selects 186, while Otsu chooses 89. This trend persists at higher threshold levels, with Kapur selecting thresholds of 49, 105, 136, and 186 at N = 5, compared to Otsu's selections of 42, 95, 140, and 170. For the Lena image, the threshold levels for both methods are more closely aligned, with Kapur selecting thresholds of 122 at N = 2, compared to Otsu's selection of 118. At N = 5, Kapur selects thresholds of 77, 121, 161, and 194, while Otsu's method chooses 44, 81, 108, and 145, with a slight preference for lower values.

Finally, for the Butterfly image, Kapur's method selects significantly different thresholds compared to Otsu's method, especially for N = 3 and above. For N = 5, Kapur selects 27, 96, 141, and 213, while Otsu's method selects 83, 106, 129, and 165, with Kapur's method consistently choosing more extreme values for the first threshold level.

This comparison shows how both methods behave differently in selecting threshold levels, with Kapur's method often choosing higher values but Otsu's method generally performing better in terms of PSNR and segmentation quality. Figure 1 shows the input images and Figures 2 and 3 illustrate segmentated images.

Image	PSNR			DB Index	
	N	Otsu Method	Kapur Method	Otsu Method	Kapur Method
Bridge	2	10.5823	9.6129	0.2259	0.2569
	3	13.8464	13.5087	0.2116	0.2112
	4	16.6289	16.236	0.2159	0.2091
	5	18.4329	18.4931	0.209	0.201
Cameraman	2	12.351	6.1718	0.5524	**0.0768**
	3	17.24	13.6257	0.1891	0.1621
	4	20.211	14.4602	0.2225	0.211
	5	21.533	20.1531	0.2445	0.2354
Lena	2	12.0374	11.8660	0.2189	0.2196
	3	15.2865	14.5629	0.2184	0.2396
	4	17.339	17.2865	0.1677	0.1902
	5	18.6581	18.1066	0.167	0.1559
Butterfly	2	10.9124	6.2464	0.2954	0.2871
	3	13.9348	8.1937	0.2154	0.1912
	4	16.9002	13.4156	0.1827	0.1735
	5	17.4215	16.7266	0.2986	0.2681
Average		**15.51**	**14.39**	**0.2396**	**0.2055**

Fig. 2. Row 1: Segmented images with Otsu's method, Row 2: Segmented images with Kapur's's method for N=5

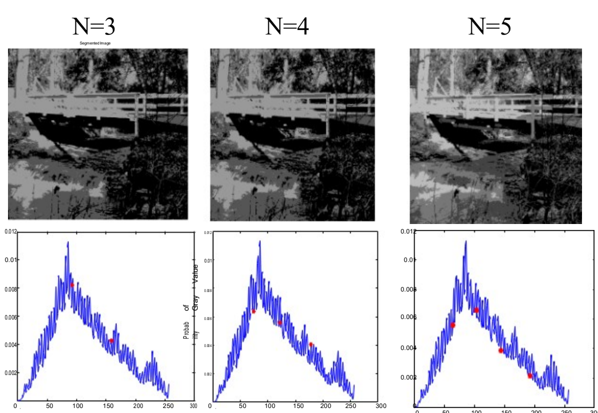


Fig. 3. Threshold Levels are shown on the histogram with reg spots in the second row of Image Bridge for threshold Levels N=3, 4, and 5 and threshold

Overall, Otsu's method performs better in terms of PSNR, while Kapur's method excels in clustering quality with lower DB Index values in some cases.

TABLE I. THRESHOLD LEVELS (TH) WITH KAPUR'S METHOD AND OTSU'S METHOD FOR THRESHOLD LEVELS N=2,3,4 AND 5 FOR FOUR IMAGES

Image	N	Th with Kapur	Th with Otsu's
Bridge	2	116	127
	3	90 161	90 157
	4	65 126 187	76 124 182
	5	53 98 147 199	65 105 144 192
Cameraman	2	186	89
	3	128 196	70 144
	4	54 113 182	59 119 156
	5	49 105 136 186	42 95 140 170
Lena	2	122	118
	3	94 155	93 121
	4	80 126 175	73 126 161
	5	77 121 161 194	44 81 108 145
	2	213	124
	3	27 213	99 151
	4	27 120 213	82 118 160
	5	27 96 141 213	83 106 129 165

VI. CONCLUSION

Image segmentation is a process in computer vision and image processing that involves dividing an image into multiple segments or regions, typically to simplify the representation of an image and make it easier to analyze. In this comparative study of image segmentation methods, Otsu's method consistently outperforms Kapur's method in terms of PSNR and DB Index across a variety of images and threshold levels. Otsu's method achieves higher PSNR values, indicating better image quality, with an average PSNR of 15.51 compared to Kapur's 14.39. The DB Index values for Otsu's method are generally lower, suggesting more effective clustering and segmentation. Specifically, Otsu's method showed superior performance on images like Bridge and Cameraman, where it produced clearer segmentation and better visual results. Kapur's method, although competitive at some threshold levels, particularly in the Lena and Butterfly images, demonstrated lower performance overall. Overall, Otsu's method proves to be a more robust and effective technique for image segmentation based on the comparative analysis of PSNR, DB Index, and threshold selection.

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