Optimizing PMU Placement in Power Systems with Heuristic Algorithms: A Modern Review

Jaydeep Parmar

Research Scholar
School of engineering
RK University
Rajkot. India
jaydeep.parmar@rku.ac.in

Dr. Chirag Parekh
School of engineering
RK University
Rajkot. India
Chiragkumar.parekh@rku.ac.in

Hardik Dahiya
School of engineering
RK University
Rajkot. India
Hardik.dahiya@rku.ac.in

Abstract—**The Phasor Measurement Unit (PMU) plays a critical role in the operation, protection, and control of modern power systems by providing real-time, synchronized measurements of bus voltage and branch current phasors. However, due to the high cost and required communication infrastructure, it is neither practical nor economical to install PMUs at every bus in the power system. The objective of the Optimal PMU Placement (OPP) problem is to determine the minimal number of PMUs required to achieve full system observability. This problem has been addressed using various techniques. Initially, graph-theoretic approaches and mathematical programming methods were proposed to solve the OPP problem and ensure system observability. Heuristic methods, which rely on experience-based strategies, offer a faster alternative for solving optimization problems where mathematical techniques cannot guarantee optimal solutions within a reasonable timeframe. This paper reviews the literature on different heuristic optimization techniques applied to the OPP problem. The methods are categorized and compared from various perspectives. Additionally, results from studies testing heuristic algorithms, both with and without considering zero-injection buses, are analyzed, demonstrating the relative advantages of the proposed heuristic approaches over one another.**

Keywords— Phasor measurement unit (PMU), Power system observability, Optimal PMU placement (OPP), Heuristic algorithm

I. INTRODUCTION

State estimation (SE) plays a critical role in monitoring and ensuring the security of power systems, particularly when accounting for contingency analysis and optimizing power flow [1]. Traditionally, remote terminal units (RTUs) have been utilized to collect key data such as real and reactive power flows, power injections, and the magnitude of bus voltages and branch currents, which are essential inputs for the supervisory control and data acquisition (SCADA) system—the primary source for state estimation [2]. However, the introduction of synchronized phasor measurements (SPM) has significantly enhanced the efficiency of power system operations, protection, and control [3]. Phasor measurement units (PMUs), which emerged in the 1990s, have become integral to wide-area monitoring systems (WAMS) by providing real-time synchronized measurements of bus voltages and branch currents [1,2]. Utilizing highly accurate synchronized signals from the global positioning system (GPS), with timing precision better than 1 millisecond, PMUs ensure precise measurements. The high sampling rates of PMUs, along with their ability to generate linear state estimators, result in a faster and more responsive voltage control system compared to traditional SCADA/EMS systems [4].

Initially, power systems were controlled using local measurements until the advent of real-time phasor measurement technology. Phasor measurements revolutionized the control of power systems by enabling the use of remote data. The key benefits of remote measurements include their ability to rapidly relay data to the controlled device and serve as feedback signals for the controller. Historically, the control process was based on local signals, local measurements, and mathematical models of external systems, such as external equivalents. By processing phasor measurements in the 0.2–2.0 Hz range, real-time state estimation of the system became possible, albeit with a slight delay. However, for effective control of power systems, measurements within the 15–60 Hz range are needed to drive the control process.

Synchrophasors offer a distinct advantage by monitoring phase angles, allowing early detection of potential instabilities and enabling the implementation of discrete switching controls to mitigate such events. Early applications of phasor measurements with continuous feedback were global in scope, addressing large-scale control challenges. For instance, HVDC controllers were used to dampen electromechanical oscillations between widely separated regions in the power grid. A model based on PMU data for small signal stability analysis was proposed in previous research. Additionally, Prony analysis has been employed for online monitoring of inter-area oscillations, while wavelet and Hilbert-Huang transforms have been applied to identify inter-area modes using PMU data.

PMUs have found applications in various aspects of power systems, including state estimation, wide-area control and monitoring, fault location and detection, wide-area

protection, transient stability analysis and prediction, thermal monitoring of transmission lines, and online steady-state angle stability monitoring. These capabilities highlight the growing importance of PMU data in enhancing the control and stability of modern power systems.

The voltage phase at a given bus, along with the current phasors from some or all of the adjacent connected lines, must be measured to accurately observe the system. Achieving a fully observable power system requires sufficient state estimation measurements, leading to the problem of optimal placement of measuring devices [20]. A system is entirely observable when PMUs are installed at all buses. However, this approach is neither cost-effective nor feasible due to the significant expense associated with PMU installations [2]. Consequently, determining the minimum number of PMUs and their optimal configuration within the system is recognized as a major challenge, commonly referred to as the Optimal PMU Placement (OPP) problem.

To address the Optimal PMU Placement (OPP) problem, various optimization techniques have been proposed in the literature, which can broadly be categorized into two main groups: conventional methods and heuristic algorithms. The first group includes approaches such as linear programming (LP), non-linear programming (NLP), dynamic programming, and combinatorial optimization. These conventional methods, though effective, often face challenges in scenarios such as PMU failure or branch outages. In contrast, advanced heuristic algorithms not only enhance system observability but also overcome many of the limitations posed by traditional techniques. These include addressing issues such as sensitivity constraints [21], communication constraints in substations [22], critical measurements [23,24], fault observability [25], and reducing mean square error (MSE) [26,27], among others.

This paper provides a review of the most widely used heuristic optimization techniques for solving the OPP problem. Section 2 outlines the formulation of the OPP problem, while Section 3 delves into the heuristic methods applied to solve OPP, along with a multidimensional comparison of the algorithms presented. Section 4 offers a detailed comparison of these methods from various perspectives, and the paper concludes with insights and recommendations in Section 5.

2. Formulation of optimal PMU placement problem

Numerical and topological observability are primary methods for evaluating system observability. Numerical observability, however, is often hindered by the complexity of matrix calculations, which limits its widespread application in observability analysis. In contrast, a system is deemed topologically observable if a spanning tree of full rank can be achieved. To enhance and streamline topological analysis, several effective rules have been developed. These rules are detailed below.

1. The voltage phasor at a bus equipped with a Phasor Measurement Unit (PMU) and the current phasors for all interconnected lines are depicted in Figure 1. These are referred to as direct measurements. Specifically, in Figure 1, bus-1 is equipped with a PMU, allowing for the measurement of the voltage phasor at bus-1 and the current phasors of the connecting branches, as outlined by Rule 1.

2. When both voltage and current phasors are known at one end of a transmission line, it ensures that the conditions at the other end of the line can be observed. This is achieved by providing the voltage phasor, as illustrated in Fig. 2. Such measurements are referred to as pseudo-measurements. In the example shown in Fig. 2, the voltage phasor at bus-1 and the current phasor of the line connecting bus-1 and bus-3 are provided. By applying Rule 2, it is then possible to determine the voltage phasor at bus-3.

3. To calculate the current phasor for a transmission line, you need the voltage phasors at both ends of the line. This method, often referred to as using pseudo-measurements, involves using the known voltage phasors at bus-2 and bus-3 (as shown in Fig. 2) to determine the current phasor for the branch connecting these buses (as illustrated in Fig. 3).
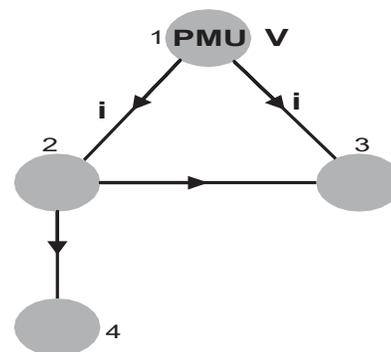

Fig. 1. PMU placement rule 1: observability with direct measurements.

4. Considering a zero-injection bus, if current phasors of all joint lines are known, except one, the current phasor of the unknown line can be calculated utilizing KCL equations. This situation is illustrated in Fig. 4. In this figure, bus-3 is zero-injection bus and current phasors of the line between bus-3, and bus-1, and also current phasor of the line between bus 3, and bus-2 is available. So by using rule 4 current phasor of the line between bus-3 and bus-4 will be known as mentioned in the fourth rule of observability rules.

5. To determine the voltage phasor of a zero-injection bus with an unknown voltage, you can use the voltage phasors of its neighbouring buses if they are known. As illustrated in Fig. 5, where bus-3 is the zero-injection bus and the voltage phasors of adjacent buses (bus-1, bus-2, and bus-4) are known, you can calculate the voltage phasor of bus-3 using node equations.

6. In scenarios where the voltage phasors of all adjacent buses to a group are known, the zero-injection buses can be determined using both Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL) equations. For instance, as illustrated in Fig. 6, if a set of zero-injection buses, such as bus-3 and bus-4, have unknown voltage phasors, but the adjacent buses (bus-1, bus-2, bus-5,

and bus-6) have known voltage phasors, then the voltage phasors of bus-3 and bus-4 can be calculated according to Rule 6.

Fig. 2 : Rule 2 for PMU placement, focusing on how to ensure the observability of bus voltage through the use of pseudo measurements.
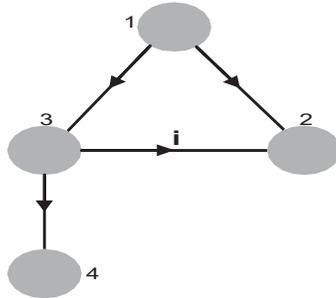

Fig 3: PMU Placement Rule 3 - Observability of Line Current $\sum_{j}^{Nbus} Sij$ through Pseudo Measurements

Various placement studies have explored the optimal placement of PMUs in power systems using different objective functions to enhance system observability. Heuristic algorithms have been employed to address objectives such as minimizing the number of PMUs, maximizing measurement redundancy, and managing contingency constraints like PMU or line outages. The following section defines each objective function relevant to solving the Optimal Placement Problem (OPP).

Minimum Number of PMUs: The primary goal of the OPP is to identify the minimum number of PMUs needed and their optimal locations to achieve complete observability of the power system. The constraint is to ensure that the power network remains fully observable. This objective function can be mathematically expressed as follows:

$$Min$$

$$s.t. \ A.S \geq I$$

$$S(i) = \begin{cases} 1: if \ bus \ i \ is \ a \ PMU \ equiped \ bus \\ 0, otherwise \end{cases}$$

$$A(i,j) = \begin{cases} 1, if \ i - j \\ 1, if \ buses \ i \ and \ j \ are \ connected \\ 0, otherwise \end{cases}$$

Another factor in addressing the OPP problem is measurement redundancy, which has been treated as an objective function in various studies. Generally, measurement redundancy refers to the number of redundant measurements taken for each bus or the frequency with which each bus is monitored, whether directly or indirectly. Therefore, it is essential to guarantee complete observability of the system.

The passage discusses the importance of measurement redundancy and contingency management in power systems. The goal is to ensure that each bus in the system has at least one redundant measurement, making it a secondary objective to maximize redundancy across the network. Additionally, contingencies such as line outages and PMU (Phasor Measurement Unit) failures are critical factors that can lead to system instability. To maintain system observability during such events, it's important to optimize the placement of a minimum number of PMUs while ensuring the system can tolerate a single line outage or PMU failure. Various heuristic methods are employed to achieve this optimal configuration.

Table 1: Test systems data

Test system	No. of branches	Number of zero injection buses	Location of zero-injection buses
IEEE_14	20	1	7
IEEE_30	41	5	6–9–11–25–28
IEEE_39	46	12	1–2–5–6–9–10–11–13–14–17–19–22
IEEE_57	78	15	4–7–11–21–22–24–26–34–36–37–39–40–45–46–48
IEEE_118	179	10	5–9–30–37–38–63–64–68–71–81

CONCLUSION

The Optimal PMU Placement (OPP) problem has been addressed using various optimization methods, categorized into two main approaches: conventional methods and heuristic algorithms. This paper presents a comprehensive literature review on heuristic algorithms, comparing their results and highlighting specific features across different studies. A detailed comparison of the results from standard test systems, such as IEEE 14-bus, 30-bus, 39-bus, 57-bus, and 118-bus, is provided. The review aims to assist researchers in exploring new concepts for solving the OPP problem. Future work will focus on developing new heuristic optimization techniques for multi-objective OPP, incorporating constraints.

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